Formerly expensive, exotic and ill-understood, today most analytic methods have—thanks to computers—become affordable and commonplace. Here is a readable review of . . .

Multivariate Analysis in Marketing

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Every discipline experiences two things during its transition from a speculative philosophy to a branch of empirical science:

1. It begins to apply and generate certain statistical techniques.
2. It seeks guidance from more mature disciplines in applying statistical models.

Both psychology and economics, for example, went through this process recently when they sought guidance from mechanics and chemistry. Unfortunately, this imitative tendency creates overabundance and overemphasis of some special statistical models. In the case of both psychology and economics, bivariate analysis received greater attention because it was all mechanics and chemistry could offer. This explains, to a large extent, relatively recent applications and development of multivariate techniques in psychology.

Currently, marketing seems to be in the transition period from speculative thinking to empirical research [96]. It is, therefore, not surprising to observe that most articles in marketing journals today either apply known statistical techniques or generate new ones especially suited to marketing research. Perhaps the greatest proliferation seems to be the application of multivariate techniques. There are two reasons. First, marketing seeks guidance from psychology and economics, both of which currently offer a variety of multivariate methods. Second, in an effort to segment markets based on individual differences, marketing researchers agree that consumer behavior and marketing environment are complex enough to make univariate or bivariate analysis inadequate. Indeed, this thinking goes so far as to state: "For the purposes of marketing research or any other applied field, most of our tools are, or should be, multivariate. One is pushed to a conclusion that unless a marketing problem is treated as a multivariate problem, it is treated superficially" [42].

In light of current interest and emphasis on multivariate analysis, this paper attempts to describe non-technically some multivariate methods, to review existing applications in marketing, and to discuss several new directions in which these methods may be potentially useful.

The following is an attempt to exhaustively review the applications in marketing of four multivariate techniques. However, only the most salient references are cited on the general description of various techniques;

some of the references contain exhaustive bibliographies on specialized techniques.

It is difficult to define multivariate analysis [62]. Broadly speaking, it includes those statistical techniques which are concerned with analyzing multiple measurements that have been made on a number of individuals [20]. In short, any simultaneous analysis of more than two variables will be part of multivariate analysis.

There exist as many or more methods in multivariate analysis as there are in both univariate and bivariate analyses. In addition, multivariate analysis possesses four distinct advantages over bivariate analysis. They are (1) economy in data collection, (2) consistency of statistical inference, (3) development of more adequate theoretical constructs, and (4) greater conceptual precision and perspective. The reader is referred to Cattell [14, 15] for a more elaborate discussion.

Multivariate techniques are classified into the following categories:

1. Tests of hypotheses about differences in means and variances on a set of variables, called "T^2" and multivariate ANOVA procedures [1, 7, 55];
2. Multiple correlation and regression [24, 31];
3. Discriminatory analysis [34, 35, 94, 112];
4. Canonical analysis [20, 53, 56, 62];
5. Principal components analysis [43, 49, 54, 70];
6. Factor analysis [9, 49, 52, 105];
7. Latent structure analysis [71, 72];
8. Cluster and profile analysis [103, 104, 107]; and
9. Multidimensional scaling [21, 111].

All of these techniques are closely related and use the same set of mathematical theorems of linear algebra. For example, suppose information has been collected from a sample of households on four socioeconomic variables: income, education, occupation, and dwelling area. If the sample consists of several distinct groups, differences can be analyzed in level and dispersion using the first set of procedures (T^2 and multivariate ANOVA). But, to predict variables from knowledge of other variables, a number of other techniques are useful. First, the dependent (criterion) variables and the independent (predictor) variables should be defined, based on some theory. If there is only one criterion variable—for example, dwelling area—then multiple regression is useful. If the criterion variable is classificatory (metropolitan vs. suburban), then discriminatory analysis is relevant. Finally, if there is more than one criterion variable—say, dwelling area and income—then canonical analysis is useful.

If the sample is to be divided into homogeneous groups based on its value on the four socioeconomic variables, profile analysis is the relevant technique. On the other hand, it might be desirable to find the underlying trait of which the four variables are indicators. In the example, such a latent variable is social class. Principal components and factor analysis are useful techniques for this purpose. If the four variables were only dichotomies latent structure analysis could be used. Finally, the dimensionality of any one of the variables might be sought because it could connote different things to different people. This would be achieved by using several multidimensional scaling techniques.

It is obvious that two or more multivariate techniques can be used on a stepwise basis. For example, do clustering of variables using factor analysis, based on which select some variables for regression analysis.

In this paper, only four of the above techniques are reviewed. They are (1) factor analysis, (2) cluster and profile analysis, (3) discriminatory analysis, and (4) canonical analysis. This restriction, despite the fact that multiple regression is extensively used—perhaps more appropriately, misused—in marketing stems from several reasons. First, these four techniques are new to marketing research, and some simple description is needed. Second, several other techniques can be submerged in any one of these four. For example, principal components analysis or latent structure analysis are, in essence, part of factor analysis. Third, the potential of these techniques is not fully diffused among the marketing scientists who have been exposed to them.

Factor Analysis

Among the multivariate techniques included here, factor analysis is most widely known and used by marketing practitioners.

"Factor analysis is basically a method for reducing a set of data into a more compact form while throwing certain properties of the data into bold relief" [76]. More technically, it is a *set* of methods in which the observable or manifest responses of individuals on a set of variables are represented as functions of a small number of latent variables called "factors." It is, therefore, an attempt to descry those hidden underlying factors which have generated the dependence or variation in the responses [80]. Such functions can be both linear and nonlinear, although generally they are limited to a linear functional relationship between the factors and the manifest responses [78]. A factor, then, is a linear combination of the variables in a data matrix. In other words,

$$F = a_1x_1 + a_2x_2 + \ldots + a_nx_n \quad (1)$$

Linear combinations are derived by using several judgmental criteria or the analytical criterion of the least squares principle. The latter suggests a close resemblance to regression. However, the peculiarity of factor analysis lies in the fact that a *number* of linear combinations each giving one factor is more common. In short, it is not at all unusual to obtain a small number of factors in any data analysis. Thus, it can be stated more generally,

$$F_m = a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \quad (2)$$

In factor analyzing a data matrix, two sets of values are obtained which are known as "factor scores" and "factor loadings." A factor score is individual i scores as a result of a linear combination of manifest scores. Thus,

$$F_i = a_1x_{1i} + a_2x_{2i} + \ldots + a_nx_{ni} \quad (3)$$

Since there are as many scores per individual i as there are linear combinations (factors), it may be generalized to,

$$F_{mi} = a_{m2}x_{1i} + a_{m2}x_{2i} + \ldots + a_{mn}x_{ni} \quad (4)$$

A factor loading is the correlation between factor scores and the manifest scores of the individuals in the sample. These correlations may be high or low, positive or negative, depending on the dependence of manifest variables and the particular method of factor analysis. A factor loading may be described as:

$$a_j = f_1x_{1j} + f_2x_{2j} + \ldots + f_Nx_{Nj} \quad (5)$$

where j = a manifest variable
N = sample size.

More generally, this is:

$$a_{mj} = f_{m1}x_{1j} + \ldots + f_{mN}x_{Nj} \quad (6)$$

Factor analysis is more appropriately a *set* of data reduction techniques rather than a single unique method. Much of the confusion in marketing literature stems from not appreciating this fact. The set is created as a result of a variety of options available to the researcher for analyzing data. These options can be grouped by (1) the nature of the data matrix to be factored, (2) the weights or coefficients to be specified in making the linear combinations, and (3) the derivation of new (rotated) factors by transformation of original factors [49, 52].

The option related to the data matrix regards factoring either a correlation matrix, a covariance matrix, or a cross-products matrix. Any multivariate analysis begins with a data matrix X consisting of n rows representing variables and N columns representing individuals. In some cases, it is advantageous to redefine rows and columns by transposing the data matrix. The cell x_{ji} refers to i individual's response on jth variable.

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This data matrix contains three types of information: level, dispersion, and shape of variables or individuals. In some analyses, all three types of information are relevant. In that case, a cross-products matrix XX' is obtained by post-multiplying the data matrix with its transpose. Each cell element contains sums of squares or cross products. If only dispersion and shape are important, levels of all variables first can be set equal, preferably at zero level, and transformed cell values can be obtained which are deviation scores. A cross-products matrix of deviation scores becomes a covariance matrix when each cell is divided by the number of individuals. Finally, if both level and dispersion are not relevant to the analysis, both of these can be equated across the variables. One way is to obtain standard scores where means are equal to zero and variances are all equal to unity. The cross-products of a matrix of standard scores then results in the well-known and common correlation matrix.

Obviously, covariance and correlation matrices are one specific way of removing the effects of level and dispersion on the data. They are commonly used because of their mathematical relationships to known distributions, such as normal distribution. However, any other method of equalizing level and dispersion would be relevant as data input for factor analysis. Of course, there are some situations where a particular type of data matrix is almost mandatory. For example, when the units of measurement of variables are diverse enough to lack common dimensionality, it is desirable that data be standardized.

On the other hand, if the researcher believes, based on his theory, that he should expect individual differences in the sample on level and dispersion, it is better to use the cross-products matrix [92, 97].

Finally, there are six separate ways that data can be correlated with the use of cross-products, covariance, or correlation procedures because, in general, there are three kinds of information: variables, people, and separate time periods. Holding one dimension constant and using the second dimension as replication, a cross-products, covariance, or correlation matrix can be obtained on the various elements of the third dimension. For example, the result could be a variable-by-variable correlation matrix, a people-by-people correlation matrix, or a time-by-time correlation matrix. Cattell [13] has given various labels to the six types. Factoring a variable-by-variable correlation matrix at a point in time is called "R-type" factor analysis, and a person-by-person correlation matrix at a point in time is called "Q-type" factor analysis.

The second option is the choice of weights for making linear combinations. This option is two-fold: judgmental methods and analytical methods. As the name implies, judgmental procedures only *approximate* some exact solutions, and there is no statistical rationale for their choice. The best known of these procedures are the centroid method, the bifactor method, and the multiple group method.

Among the analytical procedures which use the basic structure theorems of matrix algebra, the most widely known is the principal components analysis. Finally, both the judgmental and analytical procedures give further options (particularly in a correlation matrix input) to the diagonal values. For detailed descriptions of this, see Harman [49] and Horst [52].

The third option treats the derivation of new (rotated) factors by linear transformation procedures. Once again, there are two broad types: judgmental and analytical. The judgmental procedures all date back to Thurstone's simple structure principle where a factor is rotated to an extent that any one variable is highly loaded on one and only one factor. The analytical procedures use a number of variations of this simple structure principle. The two most common analytical procedures are quartimax and varimax rotations. The third option is resorted to by researchers for better interpretation of the results and has no statistical significance per se. It is also the most controversial aspect of factor analysis. For details, see Harman [49] and Horst [52].

It is obvious that possible combinations of the three types of options and further suboptions in each type generate hundreds of separate factor analyses.

Marketing Applications

Despite widespread knowledge and diffusion of factor analysis, there exists considerable confusion and heterogeneity of opinions among marketing scientists [19, 28, 29, 42, 60, 76, 90, 118]. The applications of factor analysis in marketing can be reviewed by looking at five uses of the technique.

The first use is to *infer* the dimensions which latently order products or brands in terms of preference. Data are obtained on the ranking or rating of products in terms of overall preference or liking. Then, after factor analyzing them, the researcher looks for clues *outside* the data to interpret and name the factors. The factors are, therefore, loaded with surplus meaning which adds subjectivity to the analysis.

A good example of this type of analysis comes from Stoetzel [109]. He obtained three factors based on rankings of various types of liquors (rum, whiskey, etc.) by a sample of French consumers. After looking at the factor loadings, he labeled them as sweetness, price, and regional popularity factors, based on his external knowledge about the liquor industry. Similar studies have been done on preferences for Cheshire cheeses [50], for foods [89], for television shows and magazines [25, 26, 69, 110, 119], and for brands of durable appliances [101]. The latter study reveals a good deal of brand generalization across several product classes for brands such as General Electric, Sears, Westinghouse, Hotpoint, and Frigidaire.

The problem in these studies is, of course, attributing causality to the underlying traits which are at best inferred from the researcher's own knowledge of the market outside the data. The advantages are that it is simple, assumes no knowledge on the part of the researcher, and sometimes reveals unusual things about the phenomenon under investigation.

The second use of factor analysis is to obtain structure among a set of attributes related to a product. The researcher presumably has sufficient knowledge about the product to identify its characteristics. The interest is to see which of these characteristics go together and whether they could be meaningful in terms of overall preference. Mukherjee [82], for example, collected data on a large number of scales related to coffee attributes and reduced them to a few factors which seem the underlying determinants of buyer's preference for coffee. Unlike the first use, the interpretation and labeling is limited to the data itself. Other studies of this type include obtaining patterns of attributes related to corporate character [27], to supermarket choice [32], to propensity to buy [93], to best selling books [51], and to brand choice [3]. Special mention must be made of factor analysis of semantic differentials to obtain corporate image [18].

An interesting study by Lunn [73] attempted to establish correlates of overall preference for a brand in terms of specific brand characteristics. Sheth and Ring [102], on the same lines, found that of several evaluative characteristics of a brand, only a few loaded on the same factor with the overall evaluative preference scale. This suggests that consumers prefer that brand on a very few attributes. This seems to hold for several brands in three separate product classes.

The third use of factor analysis in marketing is to cluster variables or individuals for classification and segmentation. To classify variables, the R-type factor analysis is used; to classify individuals, the Q-type factor analysis is done.

Almost a quarter of a century ago, Burt [11] showed that both the R-type and the Q-type factor analyses are equivalent *if* the starting point is a cross-products matrix. Recently, Nunnally [84] has shown how and under what circumstances the two types are not really inverse. By the same token, he shows where the inverse equivalence lies in them.

Only a handful of studies in marketing have used factor analysis for this purpose. By far the best is that of Stephenson [108] on image of public utilities based on his Q-sort technique [107]. Similarly, Clevenger, Lazier, and Clark [18] compare corporate images of a tobacco and an appliance company from two independent samples. They find good congruence between the two concepts and between the two samples. Finally, Barban and Grunbaum [4] classified ten advertising stimuli on a set of semantic differentials and found considerable differences between white and Negro respondents. It is somewhat surprising to find no use of factor analysis in the areas of pilot testing and motivation research where Q-type factor analysis seems so relevant.

The fourth use of factor analysis is to isolate, based on factor analysis of data, those variables which show greatest promise for further analysis. For example, Twedt [117] isolated three variables based on a factor analysis of 19 predictor variables (various aspects of advertisements, such as size, color, layout, etc.) and the criterion variable of readership. He then used these three variables as predictors of readership by doing multiple regression and obtained a correlation coefficient of .76, which is very high to say the least. The only other study to isolate more salient socioeconomic variables is by Sethi [95].

Finally, factor analysis is used to obtain factor scores of individuals in the sample for further analysis. Not only does this conversion of manifest scores to factor scores reduce large sets of data to a more manageable level, it also removes collinearity in the original variables. Massy [74, 75] has done the pioneering work in this direction. Others include the works of Farley [33] to explain variability in brand loyalty across products; Green, Frank, and Robinson [46] to cluster potential test market cities; and Campbell [12] to use factor scores derived from socioeconomic variables to do step-wise regressions of the size of the evoked set.

New Directions in Factor Analysis

Although factor scores have been mostly used for further regression or discriminatory analysis (in fact, the motivation to factor analyze the data is to remove collinearity which is otherwise a problem in regression), it is possible to use any other statistical technique on them, such as parametric or nonparametric tests of hypotheses about means and variances [97].

There are several major extensions of factor analysis in analyzing marketing data. Two that seem particularly relevant are discussed:

The first extension is to use factor analysis on classifactory data—dichotomies, trichotomies, etc. A large number of variables in marketing are classifactory, which makes this extension very useful and relevant. Use of factor analysis on classificatory data was first suggested by Burt [10] and Guttman [48] independently. Burt's approach seems more useful because of its close resemblance to Chi-squared analysis of manifold contingency tables.

The following is an example of the Burt approach [99]. Suppose there is information from a sample of consumers about which is their most preferred brand in several product categories. Further, suppose that our interest is in knowing the extent of brand generalization among three paper products, namely towels, napkins, and tissues. Table 1 shows the reactions of 10 consumers to various brands of paper products classified as Scott, Kleenex, and All Others.

In Table 1, *1* represents the most preferred brand and *0* represents less preferred brands. It is obvious that in a given product category, a buyer can have only one brand as the most preferred brand. The brands are, therefore, mutually exclusive and exhaustive within a product class.

Looking at the data, it is evident that the first two buyers are loyal to Scott and manifest complete brand generalization. Similarly, the fifth and sixth buyers are loyal to Kleenex. The last two buyers are not loyal to either of these two brands but could be loyal to some other brand which is part of All Others. Now, if there were complete loyalty and generalization to the three categories of brands, perfect correlation could be obtained across product classes. Furthermore, since there are three mutually exclusive and

TABLE 1

CONSUMERS

	Product	*Brand*	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*
Variables	Towels	Scott	1	1	1	1	0	0	1	1	0	0
		Kleenex	0	0	0	0	1	1	0	0	0	0
		All Others	0	0	0	0	0	0	0	0	1	1
	Napkins	Scott	1	1	1	1	0	0	0	0	0	0
		Kleenex	0	0	0	0	1	1	1	1	0	0
		All Others	0	0	0	0	0	0	0	0	1	1
	Tissues	Scott	1	1	0	0	0	0	0	0	0	0
		Kleenex	0	0	0	0	1	1	1	1	0	0
		All Others	0	0	1	1	0	0	0	0	1	1

exhaustive categories, the extent of generalization for each category can be found. In other words, for a complete generalization, there will be only three factors. On the other hand, if there is no consistent preference at all across two or more product classes for any one category, each variable will be unique, and there will be nine factors. In between, any other number of factors will be obtained depending upon the extent of brand generalization for each of the brands.

The first step is to post-multiply the data matrix by its transpose to obtain a cross-products matrix, and Table 2 gives this cross-products matrix. It will be noted that it is a square symmetric matrix and is a type of manifold contingency table. Also, it is a super-matrix in which the diagonal matrices have elements only in the diagonals. Looking at the cross-products matrix, it would appear that there should be about five factors: the first three representing complete generalization among a subset of consumers for each of the three brand categories, and the last two representing the divided loyalty between two brands, such as Scott Towel and Kleenex Napkins or Scott Towel and All Other Tissues.

Just as one can understand brand generalization, one can also understand brand switching over time within the same product class. Each time period in place of each product class now will be a classificatory variable.

A preliminary study on brand switching over time will show two things. First, that brand loyalty towards a number of brands can be measured for every consumer. In other words, brand loyalty is multidimensional in the sense that a consumer can be loyal to more than one brand (with differential degrees of loyalty).

Second, it is possible to arrive at measures of aggregate market shares of brands for the total time-period studied, where market share is a function of both the frequency of purchase and pattern of brand switching. In other words, market share is weighted by consumer's extent of loyalty which seems a more suitable measure, at least for managerial evaluation.

Another major extension of factor analysis is for obtaining functional relationships among the manifest variables [100]. The essential idea is that in time-series data (such as purchase frequency for various months of a year by a sample of households), each cell entry x_{ji} (for example, i individual's purchase frequency at jth time-period) can be considered a point on a curve where the independent axis is some unit of time and the dependent axis is the particular phenomenon under investigation with which the cell values are filled. In other words,

$$x_{ji} = f(a_i, b_i, c_i \ldots x_j) \qquad (7)$$

This functional relationship can be transformed to match the basic linear postulate of factors analysis [97, 116]. This results in the following equation:

$$x_{ji} = a_{j1}s_{1i} + \ldots + a_{jm}s_{mi} = \sum_{m=1}^{r} a_{jm}s_{mi} \qquad (8)$$

The a_{jm} (factor loadings in the normal sense) represent aggregate parameters, and s_{mi} (factor scores) represent each individual's parameters for his functional relationship. Using this approach, it is possible to do curve fitting for all linear and several nonlinear functions [100]. The parameters meet the least squares criterion. The technique, however, has several advantages over curve fitting. First, it gives parameters for each individual consumer in addition to aggregate param-

TABLE 2

		Towels			*Napkins*			*Tissues*		
		S	*K*	*A.O.*	*S*	*K*	*A.O.*	*S*	*K*	*A.O.*
Towels	Scott	6			4	2	0	2	2	2
	Kleenex		2		0	2	0	0	2	0
	All Others			2	0	0	2	0	0	2
Napkins	Scott	4	0	0	4			2	0	2
	Kleenex	2	2	0		4		0	4	0
	All Others	0	0	2			2	0	0	2
Tissues	Scott	2	0	0	2	0	0	2		
	Kleenex	2	2	0	0	4	0		4	
	All Others	2	0	2	2	0	2			4

eters. Second, several types of curves, each varying in shape for different individuals, can be estimated in one analysis. Third, a very few column vectors (two or three) summarize a large variety of individual parameters based on classification of individuals as belonging to particular functional relationships.

The functional approach has been used to obtain brand loyalty scores of individual consumers based on their frequency and pattern of purchases over a discrete time-period [98]. Obviously, in time-dependent sequences, pattern makes the difference between two individuals who have the same frequency of purchase.

Profile and Cluster Analysis

A second major multivariate technique is profile or cluster analysis. Profile analysis is a generic term for all methods concerning grouping of individuals. Cluster analysis is a generic term for all methods concerning grouping of variables. The procedures for both cluster and profile analysis are very similar; hence, both are referred to as profile analysis.

Profile analysis involves at least two separate steps. The first is the measurement of similarity between two persons or variables. The second step is classification of persons or variables based on the similarity measures.

A series of cut-and-try methods have been proposed to perform profile analysis [2, 22, 85, 86, 103, 104]. Most of these calculate distance between two persons by putting them in some sort of space. In general, a person with his scores on n variables is considered a point in n-dimensional space. The distance between two points gives a measure of similarity: The greater the distance, the less similar the two points. Then, several arbitrary methods are available which specify the cut-off point for the marginal person to be included in a group [103, 104].

The two most common distance measures are calculation of absolute differences and distances based on Pythagorean theorem. Mathematically, they can be stated as:

$$d_{ij} = \sum_{k=1}^{n} | a_{ik} - a_{jk} | \tag{9}$$

and

$$d_{ij} = \left[\sum_{k=1}^{n} (a_{ik} - a_{jk})^2 \right]^{1/2} \tag{10}$$

where i and j are two persons or points in n-dimensional space constructed from measurements on k scales.

The similarity measure d_{ij} contains all the three types of information: level, dispersion, and shape. By removing one or more of these, several other distance measures are possible. Recently, Sokal and Sneath [104] have suggested several procedures for classification under the name of numerical taxonomy. For an excellent description and review of these, see Frank and Green [36].

Most of the profile analyses suffer from two problems. First is the lack of invariance of similarity between two persons resulting from adding or dropping the dimensions on which they are measured. This becomes a serious issue when the dimensions for comparison are based on convenience and, at best, judgment instead of any theory. The second, a related problem, is the orthogonality of dimensions. If the dimensions are not orthogonal, the distances based on Euclidian space become less meaningful.

Except for the graphic representation of semantic differential scores [79] as profiles, the application of profile analysis is very recent in marketing. The first attempts relate to measurements of similarity between self-concept and some consumer behavior variable where attempt is made to show greater congruence between the two [6, 47]. However, classification of consumers in these studies is already known, based on the particular consumer behavior under investigation.

The pioneering efforts to measure similarity and then classify objects or people come from Green, Frank, and Robinson [46]. They used the distances to obtain clusters of cities which are potential for test marketing. A good review on classification procedures recently appeared in *Applied Statistics* [61]. The only other research analysis in this area comes from Myers and Nicosia [83], based on Tryon's conceptual and computerized methods of cluster analysis [113, 114, 115].

New Directions

Despite its simplicity, profile analysis is not often used in marketing. It does seem potentially relevant in several areas of marketing outside consumer behavior per se. For example, Reilly's law of retail gravitation can be more accurately represented in today's economy by taking a set of forces (brand choices in the store, pricing, parking convenience, check-out counter facilities, convenience and ease of in-store shopping, cleanliness of the store, etc.) on which the stores can be compared in terms of similarity.

From this, a clustering of stores in a specific region can be obtained. Another example is the evaluation and promotion of brands based on their similarity at the market place. For example, a company such as General Foods may cluster various types of frozen juices based on the evaluative judgments of the consumer and the retailer.

Profile analysis, however, does not lend itself to algebra, and, hence, analytical solutions. On the other hand, due to the level of maturity of the discipline, analytical solutions are more desirable. There are two directions in which some of the arbitrariness in profile analysis can be eliminated.

First, it is possible to factor analyze a similarity matrix D obtained from the various d_{ij} values. The factors then will cluster individuals on analytical grounds [57]. Second, it is possible to combine both the steps in profile analysis and obtain an analytical solution if a cross-products matrix of individuals is factor analyzed.

Discriminant Analysis

Discriminant analysis is useful in situations where a total sample is divided into known groups based on some classificatory variable, and the researcher is interested in understanding group differences or in predicting correct belonging to a group of a new sample based on the information on a set of predictor variables.

Discriminant analysis, therefore, can be considered either a type of profile analysis or a type of multiple regression. As a profile analysis, its significance lies in the structure of weights obtained which discriminate various groups. Then, it is sometimes referred to as structural analysis [68]. As a multiple regression, its significance lies in providing predictive power to the researcher in terms of classifying individuals more accurately than by chance. In either case, the criterion variable is single and classificatory.

Discriminant analysis entails transformation of scores of individuals on a set of predictor variables by using a set of linear weights. The transformed value is called the discriminant score. This score is treated as projection of a point on the discriminant axis and, depending on whether it lies above or below the discriminant line, the individual is classified as belonging to one or the other group. The linear transformation of raw scores into discriminant scores can be represented as:

$$y_i = a_1x_{1i} + a_2x_{2i} + \ldots a_nx_{ni} \qquad (11)$$

where y_i = individual i's discriminant score

x_{ji} = individual i's score on jth variable

The procedure, therefore, would appear similar to factor analysis. However, the analytical procedure of obtaining optimum weights (a_k) is based on the principle of maximizing the ratio:

$$\frac{\text{Variance between means on y}}{\text{Variance within groups on y}}$$

This is analogous to one-way classification in analysis of variance [34].

It is possible to obtain more than one discriminant axis similar to factor analysis. However, the total number of axes do not exceed the number of groups minus one. In a two-group situation, therefore, only one discriminant axis and one discriminant score for each individual are obtained.

Once the discriminant axis is obtained, the function could be tested for significance. Then, based on the discriminant scores, individuals in the sample would be classified in one of the groups. The proportion of correct classification then is compared against what could have been predicted by chance without any knowledge of the scores on the predictor variables. To this extent, it resembles the Bayesian approach. It is, however, more appropriate to validate the analysis by using the discriminant weights on another sample of individuals because predicting on the same sample from which coefficients are derived is shown to result in biases [39].

A large number of research studies in marketing have recently applied discriminant analysis, mostly for prediction purposes. Evans [30], for example, attempted to discriminate new Ford and Chevrolet buyers based on personality needs, socioeconomic variables, and a combination of both, with little success. Recently, however, Ito [58] successfully discriminated loyal and switching Ford and Chevrolet buyers on the basis of nine attitude scales. However, he used intention measures for the second purchase as opposed to actual purchases. A number of studies [23, 37, 38, 64, 65, 66, 67, 88, 91] deal with prediction of innovators from non-adoptors or late adoptors on a series of socioeconomic, personality, psychological, and purchase characteristics. The success is only moderate.

Other areas of applications include discriminating among listeners who sent for a program guide from those who did not [77], among various types of holders of savings accounts [17], among consumer decision types on personality variables [8], among social classes [41], among those who intend to buy [87], and to obtain scale values in an advertising study [5]. Perhaps the most extensive use of discriminant analysis in a single study comes from Sethi [95], who attempted to discriminate high and low buyers of brands of analgesics on socioeconomic and purchase characteristics and a combination of both types of variables.

In using discriminant analysis for prediction purposes, there are two problems a researcher generally encounters. First, the dependent classificatory variable is often forced on the data by the researcher with the result that there exists an overlap between groups to an extent that separation of groups is not powerful. A greater caution is needed among the users of discriminant analysis in their definition of classificatory dependent variable [68, 97]. The second problem concerns validation of the analysis pointed out earlier. If the same sample is used, it overestimates the predictive power of the discriminant function [39]. A pragmatic suggestion is to split the sample into half, using one half for analysis and the other half for validation.

The above-mentioned problems have led King [68] to suggest that discriminant analysis be used not so much for prediction as for relative importance of predictor variables based on structural analysis of discriminant coefficients.

With sufficient care, a number of exciting applications of discriminant analysis can be made in marketing. For example, it is suggested that various marketing strategies may be considered "experiments," which could be evaluated on a series of performance measures. Each experiment would be a group, and performance measures would be various predictor variables. Similarly, various advertising campaigns can be treated as experiments, then evaluated. Not only will this show the relative effectiveness of various strategies, but it will also show the relative effects on various performance measures [97].

Another important new direction is the extension of discriminant analysis in situations where the predictor variables are also classificatory (only binary, though). Clarigbold [16] gives a good description of this. This extension is quite useful in view of the fact that a large number of variables—such as sex, religion, occupation, etc.—are classificatory. It is somewhat surprising that Fisher [34] himself suggested such extension, and still it is not widely known in marketing [59].

Canonical Analysis

Canonical analysis is an extension of multiple regression to a situation where there is more than one criterion variable. There are, then, two sets of measurements: a criterion set and a predictor set, which have to be correlated. The underlying principle is to create a linear combination of each set of variables for each individual by obtaining a set of weights which maximizes the correlation between the two sets.

Canonical analysis is appropriate when the researcher is more interested in the *overall* relation between the predictor and the criterion sets. He then may isolate those predictor variables which contribute most to this overall relation. In addition, he also may note which of the criterion variables are more affected.

Canonical analysis is scarcely known to marketing researchers. Otherwise, there are many areas of consumer research, advertising research, and product research where it is most appropriate but no study seems to exist. The recent summary of the technique by Green, Halbert, and Robinson [45] should enable marketing researchers to more fully appreciate the technique. The only study in which its use was made is by Kernan [63]. He obtained an overall relation between a set of choice decision behaviors and a set of personality variables.

In consumer behavior, most of the variables of interest to marketing practitioners have multi-attributes. For example, brand loyalty is indicated by probability of purchase, time interval between purchases, and magnitude of buying. Consumer liking of a brand is likely to be based on a number of brand attributes. Thus, to measure effectiveness of a set of controllable variables, cononical analysis is relevant since effects are numerous.

In summary, then, there are two possible extensions of the technique which may prove useful in marketing. The first is to perform canonical analysis on a cross-products matrix similar to factor analysis. Since the input data matrix retains information on level and dispersion, it may prove more appropriate when it is known that individual variability is likely to be high. The second suggestion is to extend the technique to binary data.

This review of multivariate analysis of marketing data hints at several possible extensions of the techniques.

Multivariate techniques are extremely useful in marketing, and with the availability of canned computer programs, it seems inevitable that their use will increase in marketing and advertising research.

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